$$\frac{9!}{5!(9-5)!}a^4b^5 = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 4!}a^4b^5$$
$$= \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2 \cdot 1}a^4b^5 \qquad \text{Divide out 5!.}$$
$$= 126a^4b^5$$

EXAMPLE 9 Find the third term of the expansion of $(3x - 2y)^6$.

Solution We begin by finding the third term in the expansion of $(a + b)^6$.

$$\textbf{2.}\quad \frac{6!}{2!(6-2)!}a^4b^2 = \frac{6 \cdot 5 \cdot 4!}{2 \cdot 1 \cdot 4!}a^4b^2 = 15a^4b^2$$

We can then substitute $3x$ for a and $-2y$ for b in Equation 2 to obtain the third term of the expansion of $(3x - 2y)^6$.

$$15a^4b^2 = 15(3x)^4(-2y)^2$$
$$= 15(3)^4(-2)^2x^4y^2$$
$$= 4860x^4y^2$$

9.1 EXERCISES

In Exercises 1–12, evaluate each expression.

1. $4!$ **2.** $-5!$ **3.** $3! \cdot 6!$ **4.** $0! \cdot 7!$

5. $6! + 6!$ **6.** $5! - 2!$ **7.** $\dfrac{9!}{12!}$ **8.** $\dfrac{8!}{5!}$

9. $\dfrac{5!7!}{9!}$ **10.** $\dfrac{3!5!7!}{1!8!}$ **11.** $\dfrac{18!}{6!(18-6)!}$ **12.** $\dfrac{15!}{9!(15-9)!}$

In Exercises 13–28, use the binomial theorem to expand each binomial.

13. $(a + b)^3$ **14.** $(a + b)^4$ **15.** $(a - b)^5$ **16.** $(x - y)^4$

17. $(2x + y)^3$ **18.** $(x + 2y)^3$ **19.** $(x - 2y)^3$ **20.** $(2x - y)^3$

21. $(2x + 3y)^4$ **22.** $(2x - 3y)^4$ **23.** $(x - 2y)^4$ **24.** $(x + 2y)^4$

25. $(x - 3y)^5$ **26.** $(3x - y)^5$ **27.** $\left(\dfrac{x}{2} + y\right)^4$ **28.** $\left(x + \dfrac{y}{2}\right)^4$

In Exercises 29–48, find the required term in each binomial expansion.

29. $(a + b)^4$; 3rd term **30.** $(a - b)^4$; 2nd term **31.** $(a + b)^7$; 5th term **32.** $(a + b)^5$; 4th term

33. $(a - b)^5$; 6th term **34.** $(a - b)^8$; 7th term **35.** $(a + b)^{17}$; 5th term **36.** $(a - b)^{12}$; 3rd term

37. $(a - \sqrt{2})^4$; 2nd term **38.** $(a - \sqrt{3})^8$; 3rd term **39.** $(a + \sqrt{3}b)^9$; 5th term **40.** $(\sqrt{2}a - b)^7$; 4th term

41. $\left(\dfrac{x}{2} + y\right)^4$; 3rd term **42.** $\left(m + \dfrac{n}{2}\right)^8$; 3rd term **43.** $\left(\dfrac{r}{2} - \dfrac{s}{2}\right)^{11}$; 10th term **44.** $\left(\dfrac{p}{2} - \dfrac{q}{2}\right)^9$; 6th term

9.2 EXERCISES

In Exercises 1–2, write the first six terms of the sequence defined by each function.

1. $f(n) = 5n(n - 1)$

2. $f(n) = n\left(\frac{n-1}{2}\right)\left(\frac{n-2}{3}\right)$.

In Exercises 3–8, find the next term of each sequence.

3. 1, 6, 11, 16 . . .

4. 1, 8, 27, 64, . . .

5. $a, a + d, a + 2d, a + 3d, \ldots$.

6. $a, ar, ar^2, ar^3, \ldots$.

7. 1, 3, 6, 10, . . .

8. 20, 17, 13, 8, . . .

In Exercises 9–16, find the sum of the first five terms of the sequence with the given general term.

9. n

10. $2k$

11. 3

12. $4k^0$

13. $2\left(\frac{1}{3}\right)^n$

14. $(-1)^n$

15. $3n - 2$

16. $2k + 1$

In Exercises 17–24, a sequence is defined recursively. Find the first four terms of each sequence.

17. $a_1 = 3$ and $a_{n+1} = 2a_n + 1$

18. $a_1 = -5$ and $a_{n+1} = -a_n - 3$

19. $a_1 = -4$ and $a_{n+1} = \frac{a_n}{2}$

20. $a_1 = 0$ and $a_{n+1} = 2a_n^2$

21. $a_1 = k$ and $a_{n+1} = a_n^2$

22. $a_1 = 3$ and $a_{n+1} = ka_n$

23. $a_1 = 8$ and $a_{n+1} = \frac{2a_n}{k}$

24. $a_1 = m$ and $a_{n+1} = \frac{a_n^2}{m}$

In Exercises 25–28, tell whether each series is an alternating infinite series.

25. $-1 + 2 - 3 + \cdots + (-1)^n n + \cdots$

26. $a + \frac{a}{b} + \frac{a}{b^2} + \cdots + a\left(\frac{1}{b}\right)^{n-1} + \cdots; b = 4$

27. $a + a^2 + a^3 + \cdots + a^n + \cdots; a = 3$

28. $a + a^2 + a^3 + \cdots + a^n + \cdots; a = -2$

In Exercises 29–42, evaluate each sum.

29. $\sum_{k=1}^{5} 2k$

30. $\sum_{k=3}^{6} 3k$

31. $\sum_{k=3}^{4} (-2k^2)$

32. $\sum_{k=1}^{100} 5$

33. $\sum_{k=1}^{5} (3k - 1)$

34. $\sum_{n=2}^{5} (n^2 + 3n)$

35. $\sum_{k=1}^{1000} \frac{1}{2}$

36. $\sum_{x=4}^{5} \frac{2}{x}$

37. $\sum_{x=3}^{4} \frac{1}{x}$

38. $\sum_{x=2}^{6} (3x^2 + 2x) - 3\sum_{x=2}^{6} x^2$

39. $\sum_{x=1}^{4} (4x + 1)^2 - \sum_{x=1}^{4} (4x - 1)^2$

40. $\sum_{x=0}^{10} (2x - 1)^2 + 4\sum_{x=0}^{10} x(1 - x)$

41. $\sum_{x=6}^{8} (5x - 1)^2 + \sum_{x=6}^{8} (10x - 1)$

42. $\sum_{x=2}^{7} (3x + 1)^2 - 3\sum_{x=2}^{7} x(3x + 2)$

9.3 EXERCISES

In Exercises 1–6, write the first six terms of the arithmetic sequences with the given properties.

1. $a = 1; d = 2$

2. $a = -12; d = -5$

3. $a = 5$; 3rd term is 2

4. $a = 4$; 5th term is 12

5. 7th term is 24; common difference is $\frac{5}{2}$.

6. 20th term is -49; common difference is -3.

In Exercises 7–10, find the sum of the first n terms of each arithmetic sequence.

7. $5 + 7 + 9 + \cdots$ (to 15 terms)

8. $-3 + (-4) + (-5) + \cdots$ (to 10 terms)

9. $\sum_{n=1}^{20} \left(\frac{3}{2}n + 12\right)$

10. $\sum_{n=1}^{10} \left(\frac{2}{3}n + \frac{1}{3}\right)$

11. Find the sum of the first 30 terms of an arithmetic sequence with 25th term of 10 and a common difference of $\frac{1}{2}$.

12. Find the sum of the first 100 terms of an arithmetic sequence with 15th term of 86 and first term of 2.

13. If the fifth term of an arithmetic sequence is 14 and its second term is 5, find the 15th term.

14. Can an arithmetic sequence have a first term of 4, a 25th term of 126, and a common difference of $4\frac{1}{4}$? Explain.

15. Insert three arithmetic means between 10 and 20.

16. Insert five arithmetic means between 5 and 15.

17. Insert four arithmetic means between -7 and $\frac{2}{3}$.

18. Insert three arithmetic means between -11 and -2.

In Exercises 19–26, write the first four terms of each geometric sequence with the given properties.

19. $a = 10; r = 2$

20. $a = -3; r = 2$

21. $a = -2$ and $r = 3$

22. $a = 64; r = \frac{1}{2}$

23. $a = 3; r = \sqrt{2}$

24. $a = 2; r = \sqrt{3}$

25. $a = 2$; 4th term is 54

26. 3rd term is 4; $r = \frac{1}{2}$

In Exercises 27–32, find the sum of the indicated terms of each geometric sequence.

27. $4, 8, 16, \ldots$ (to 5 terms)

28. $9, 27, 81, \ldots$ (to 6 terms)

29. $2, -6, 18, \ldots$ (to 10 terms)

30. $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, \ldots$ (to 12 terms)

31. $\sum_{n=1}^{6} 3\left(\frac{3}{2}\right)^{n-1}$

32. $\sum_{n=1}^{6} 12\left(-\frac{1}{2}\right)^{n-1}$

In Exercises 33–36, find the sum of each infinite geometric sequence.

33. $6 + 4 + \frac{8}{3} + \cdots$

34. $8 + 4 + 2 + 1 + \cdots$

35. $\sum_{n=1}^{\infty} 12\left(-\frac{1}{2}\right)^{n-1}$

36. $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^{n-1}$

37. Insert three positive geometric means between 10 and 20.

38. Insert five geometric means between -5 and 5, if possible.