Radioactive Decay

A decreasing function determined by the equation of the form $y = ab^{kt}$ models a process called **radioactive decay**. The atomic structure of a radioactive material changes as the material emits radiation. Uranium, for example, changes (decays) into thorium, then into radium, and eventually into lead.

Experiments have determined the time it takes for half of a sample of a given radioactive material to decompose. This time is a constant, called the material's **half-life**. The amount of radioactive material present in a radioactive material decays exponentially according to the following model.

Radioactive Decay Formula

If A is the amount of radioactive material present at time t, A_0 was the amount present at $t = 0$, and h is the material's half-life, then

$$A = A_0 2^{-t/h}$$

EXAMPLE 5 The half-life of radium is approximately 1600 years. How much of a 1-gram sample will remain after 660 years?

Solution In this example, $A_0 = 1$, $h = 1600$, and $t = 660$. We substitute these values into the formula for radioactive decay and simplify.

$$A = A_0 2^{-t/h}$$
$$A = 1 \cdot 2^{-660/1600}$$
$$\approx 0.751320306 \quad \text{Use a calculator.}$$

After 660 years, approximately 0.75 gram of radium will remain. ■

Compound Interest

An example of exponential growth is **compound interest**. If interest earned on money in a bank account is allowed to accumulate in the account, that interest will also earn interest. The balance in such an account will grow exponentially according to the following model.

Compound Interest Formula

If A_0 dollars are deposited in an account earning an annual rate r, compounded k times per year, then the amount A in the account after t years is given by

$$A = A_0\left(1 + \frac{r}{k}\right)^{kt}$$

EXAMPLE 6 In the name of a newborn child, a parent deposits \$8000 in a savings plan that earns 9% interest, compounded quarterly. If the money is left untouched, how much will the child have in 55 years?

Solution We can substitute 8000 for A_0, 0.09 for r, and 55 for t into the formula for compound interest and calculate A. Because interest is paid quarterly, $k = 4$.

We substitute 5 for x and calculate I.

$$I = 12(0.6)^5$$
$$I \approx 0.93312$$

At a depth of 5 meters, the intensity of the light is slightly less than 1, or about one-twelfth of the intensity at the surface. ■

Watching Money Grow

EXAMPLE 8 If \$1 is deposited in an account earning 9% annual interest, compounded monthly, estimate how much will be in the account in 70 years.

Solution We can substitute 1 for A_0, .09 for r, and 12 for k into the formula

$$A = A_0\left(1 + \frac{r}{k}\right)^{kt}$$

and simplify to get

$$A = (1.0075)^{12t}$$

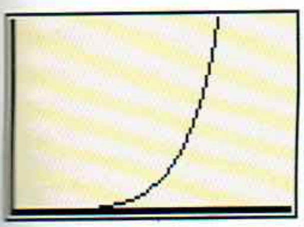


FIGURE 6-5

We now use a graphing calculator to see how the money grows year by year. We graph the function $A = (1.0075)^{12t}$ in the viewing window $0 \le t \le 100$ and $0 \le A \le 750$ to obtain the graph shown in Figure 6-5. We can then use the TRACE and ZOOM features to estimate that \$1 grows to the surprising amount of approximately \$532 in 70 years. ■

6.1 EXERCISES

In Exercises 1–4, find each value to four decimal places.

1. $4^{\sqrt{3}}$
2. $5^{\sqrt{2}}$
3. 7^{π}
4. $3^{-\pi}$

In Exercises 5–12, graph each exponential function.

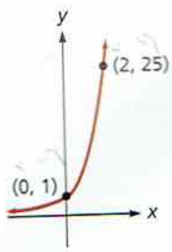
5. $y = 3^x$
6. $y = 5^x$
7. $y = \left(\frac{1}{5}\right)^x$
8. $y = \left(\frac{1}{3}\right)^x$
9. $y = -2^x$
10. $y = -3^x$
11. $y = \left(\frac{3}{4}\right)^x$
12. $y = \left(\frac{4}{3}\right)^x$

In Exercises 13–28, graph each function.

13. $y = 3^x - 1$
14. $y = 2^x + 3$
15. $y = 2^x + 1$
16. $y = 4^x - 4$
17. $y = 3^{x-1}$
18. $y = 2^{x+3}$
19. $y = 3^{x+1}$
20. $y = 2^{x-3}$
21. $y = 2^{x+1} - 2$
22. $y = 3^{x-1} + 2$
23. $y = 3^{x-2} + 1$
24. $y = 3^{x+2} - 1$
25. $y = 5(2^x)$
26. $y = 2(5^x)$
27. $y = 3^{-x}$
28. $y = 2^{-x}$

In Exercises 29–36, find the value of b, if any, that would cause the graph of $y = b^x$ to look like the graph indicated.

29.



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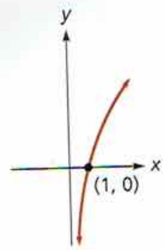

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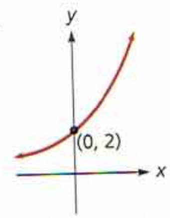

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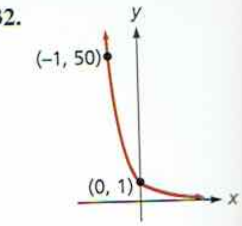

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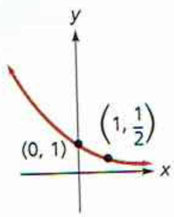

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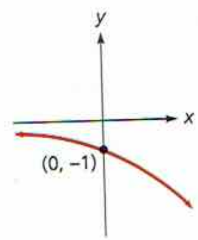

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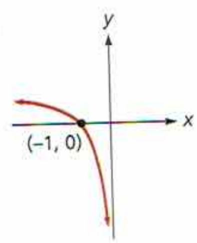

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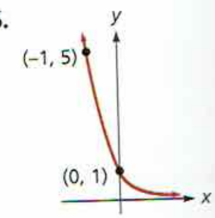

In Exercises 37–42, solve each problem.

37. Tritium decay Tritium, a radioactive isotope of hydrogen, has a half-life of 12.4 years. Of an initial sample of 50 grams, how much will remain after 100 years?

38. Carbon-14 decay The half-life of radioactive carbon-14 is 5700 years. How much of an initial sample will remain after 3000 years?

39. Radioactive decay A radioactive material decays according to the formula $A = A_0(\frac{2}{3})^t$, where A_0 is the amount present initially and t is the time in years. Find the amount that will be present in 5 years.

40. Plutonium decay One of the isotopes of plutonium, ^{237}Pu, decays with a half-life of 40 days. How much of an initial sample will remain after 60 days?

41. Californium decay The half-life of one isotope of californium, ^{253}Cf, is 18 days. The half-life of another, ^{254}Cf, is 60 days. If 1 gram of each is present initially, find the amount of ^{254}Cf present when 0.5 gram of ^{253}Cf remains. (*Hint:* One-half of the ^{253}Cf remains after one half-life.)

42. Comparing radioactive decay One isotope of holonium, ^{162}Ho, has a half-life of 22 minutes. The half-life of a second isotope, ^{164}Ho, is 37 minutes. Starting with a sample containing equal amounts, find the ratio of the amounts of ^{162}Ho to ^{164}Ho after one hour.

In Exercises 43–48, assume there are no deposits or withdrawals.

43. Compound interest An initial deposit of \$500 earns 8% interest, compounded quarterly. How much will be in the account in 10 years?

44. Compound interest An initial deposit of \$1000 earns 9% interest, compounded monthly. How much will be in the account in $4\frac{1}{2}$ years?

45. Comparing interest rates How much more interest could \$500 earn in 5 years, compounded quarterly, if the annual interest rate were $5\frac{1}{2}$% instead of 5%?

46. Comparing savings plans One bank guarantees to pay interest at 7.25%, compounded monthly. Another bank offers 7.35%, compounded annually. Which bank provides the better investment?

47. Compound interest If \$1 had been invested on July 4, 1776, at 5% interest, compounded annually, what would it be worth on July 4, 2076?

27. **World population growth** The population of the earth is approximately 5.2 billion people and is growing at an annual rate of 1.9%. Assuming a Malthusian growth model, find the world population in 30 years.

28. **World population growth** Assuming a Malthusian growth model, find the world population in 40 years. (See Exercise 27.)

29. **World population growth** Assuming a Malthusian growth model, by what factor will the current population of the earth increase in 50 years? (See Exercise 27.)

30. **World population growth** Assuming a Malthusian growth model, by what factor will the current population of the earth increase in 100 years? (See Exercise 27.)

31. **Growth of a nation** A country with a population of 2×10^5 people is expected to double every 20 years. Assuming a Malthusian model, find the population in 35 years.

32. **Town planning** The population of a small town is now 140 persons and is expected to grow exponentially, tripling every 15 years. Assuming a Malthusian model, what do the city planners project the population to be in 5 years?

33. **Medicine** The concentration, x, of a certain drug in an organ after t minutes is given by $x = 0.08(1 - e^{-0.1t})$. Find the concentration of the drug after 30 minutes.

34. **Medicine** Refer to Exercise 33. Find the initial concentration of the drug.

35. **Epidemics** Refer to Example 6. How many people will have the HIV virus in 5 years?

36. **Epidemics** Refer to Example 6. How many people will have the HIV virus in 10 years?

37. **Drug absorption** The amount A of a drug remaining in a person's bloodstream after t hours is given by the formula

$$A = A_0e^{kt}$$

where A_0 is the initial dose. After 2.3 hours, one-half of an initial dose of triazolam (a drug for treating insomnia) will remain. What percent will remain after 24 hours?

38. **Skydiving** Before the parachute opens, a skydiver's velocity v (in meters per second) is given by $v = 50(1 - e^{-0.2t})$. Find the initial velocity.

39. **Skydiving** Refer to Exercise 38 and find the velocity after 20 seconds.

40. **Free-falling objects** After t seconds, a certain falling object has a velocity v given by $v = 50(1 - e^{-0.3t})$. Which is falling faster after 2 seconds, this object or the skydiver in Exercise 38?

41. If $e^{t+3} = ke^t$, find k.

42. If $e^{3t} = k^t$, find k.

In Exercises 43–44, use a graphing calculator to solve each problem.

43. In Example 7, suppose that better farming methods change the formula for food growth to $y = 31x + 2000$. How long will the food supply be adequate?

44. In Example 7, suppose that a birth-control program changed the formula for population growth to $P = 1000e^{0.01t}$. How long will the food supply be adequate?

45. The value of e can be calculated to any degree of accuracy by adding the first several terms of the following list:

$$1, 1, \frac{1}{2}, \frac{1}{2 \cdot 3}, \frac{1}{2 \cdot 3 \cdot 4}, \frac{1}{2 \cdot 3 \cdot 4 \cdot 5}, \quad \text{and so on}$$

The more terms that are added, the closer the sum will be to e. Add the first eight numbers in the preceding list. To how many decimal places is the sum accurate

46. Graph the function defined by the equation

$$y = f(x) = \frac{e^x + e^{-x}}{2}$$

from $x = -2$ to $x = 2$. The graph will look li[ke a] parabola, but it is not. The graph, called a **catenar[y]**, important in the design of power distribution netw[orks] because it represents the shape of a uniform fle[xible] cable whose ends are suspended from the same he[ight]. The function is called the **hyperbolic cosine func[tion]**