Because 0 and 2 are acceptable values of x, each number is a root of the original equation. Verify this by checking each root. ■

Formulas

Many formulas involve second-degree equations. For example, if an object is fired straight up into the air with an initial velocity of 88 feet per second, its height is given by the formula $h = 88t - 16t^2$, where h represents height (in feet) and t represents elapsed time (in seconds) since it was fired.

To solve this formula for t, we use the quadratic formula.

$$h = 88t - 16t^2$$

$$16t^2 - 88t + h = 0 \quad \text{Add } 16t^2 \text{ and } -400t \text{ to both sides.}$$

$$t = \frac{-(-88) \pm \sqrt{(-88)^2 - 4(16)(h)}}{2(16)} \quad \text{Substitute into the quadratic formula.}$$

$$t = \frac{88 \pm \sqrt{7744 - 64h}}{32} \quad \text{Simplify.}$$

2.3 EXERCISES

In Exercises 1–12, solve each equation by factoring. Check all answers.

1. $x^2 - x - 6 = 0$ **2.** $x^2 + 8x + 15 = 0$ **3.** $x^2 - 144 = 0$ **4.** $x^2 + 4x = 0$

5. $2x^2 + x - 10 = 0$ **6.** $3x^2 + 4x - 4 = 0$ **7.** $5x^2 - 13x + 6 = 0$ **8.** $2x^2 + 5x - 12 = 0$

9. $15x^2 + 16x = 15$ **10.** $6x^2 - 25x = -25$ **11.** $12x^2 + 9 = 24x$ **12.** $24x^2 + 6 = 24x$

In Exercises 13–20, use the square root property to solve each equation. You may need to factor an expression.

13. $x^2 = 9$ **14.** $x^2 = 20$ **15.** $y^2 - 50 = 0$ **16.** $x^2 - 75 = 0$

17. $(x - 1)^2 = 4$ **18.** $(y + 2)^2 - 49 = 0$ **19.** $a^2 + 2a + 1 = 9$ **20.** $x^2 - 6x + 9 = 25$

In Exercises 21–32, complete the square to make each binomial a perfect trinomial square.

21. $x^2 + 6x$ **22.** $x^2 + 8x$ **23.** $x^2 - 4x$ **24.** $x^2 - 12x$

25. $a^2 + 5a$ **26.** $t^2 + 9t$ **27.** $r^2 - 11r$ **28.** $s^2 - 7s$

29. $y^2 + \frac{3}{4}y$ **30.** $p^2 + \frac{3}{2}p$ **31.** $q^2 - \frac{1}{5}q$ **32.** $m^2 - \frac{2}{3}m$

In Exercises 33–44, solve each equation by completing the square.

Exercises 45–56, use the quadratic formula to solve each equation.

5. $x^2 - 12 = 0$
46. $x^2 - 20 = 0$
47. $2x^2 - x - 15 = 0$
48. $6x^2 + x - 2 = 0$
9. $5x^2 - 9x - 2 = 0$
50. $4x^2 - 4x - 3 = 0$
51. $2x^2 + 2x - 4 = 0$
52. $3x^2 + 18x + 15 = 0$
3. $-3x^2 = 5x + 1$
54. $2x(x + 3) = -1$
55. $5x\left(x + \frac{1}{5}\right) = 3$
56. $7x^2 = 2x + 2$

n Exercises 57–64, use the discriminant to determine the nature of the roots of each equation. ***Do not solve the equations.***

57. $x^2 + 6x + 9 = 0$
58. $x^2 - 5x + 2 = 0$
59. $3x^2 - 2x + 5 = 0$
60. $9x^2 + 42x + 49 = 0$
61. $10x^2 + 29x = 21$
62. $10x^2 + x = 21$
63. $-3x^2 + 2x = 21$
64. $-8x^2 - 2x = 13$
65. Find two values of k so that $x^2 + kx + 3k - 5 = 0$ will have two roots that are equal.
66. For what value(s) of b will the solutions of $x^2 - 2bx + b^2 = 0$ be equal?
67. Does $1492x^2 + 1984x - 1776 = 0$ have any roots that are real numbers?
68. Does $2004x^2 + 10x + 1994 = 0$ have any roots that are real numbers?

In Exercises 69–86, change each equation to quadratic form and solve by any method.

69. $x + 1 = \frac{12}{x}$
70. $x - 2 = \frac{15}{x}$
71. $8x - \frac{3}{x} = 10$
72. $15x - \frac{4}{x} = 4$
73. $\frac{5}{x} = \frac{4}{x^2} - 6$
74. $\frac{6}{x^2} + \frac{1}{x} = 12$
75. $x\left(30 - \frac{13}{x}\right) = \frac{10}{x}$
76. $x\left(20 - \frac{17}{x}\right) = \frac{10}{x}$
77. $(a - 2)(a + 4) = 2a(a - 3)$
78. $\frac{4 + a}{2a} = \frac{a - 2}{3}$
79. $\frac{1}{x} + \frac{3}{x + 2} = 2$
80. $\frac{1}{x - 1} + \frac{1}{x - 4} = \frac{5}{4}$
81. $\frac{1}{x + 1} + \frac{5}{2x - 4} = 1$
82. $\frac{x(2x + 1)}{x - 2} = \frac{10}{x - 2}$
83. $x + 1 + \frac{x + 2}{x - 1} = \frac{3}{x - 1}$
84. $\frac{1}{4 - y} = \frac{1}{4} + \frac{1}{y + 2}$
85. $\frac{24}{a} - 11 = \frac{-12}{a + 1}$
86. $\frac{36}{b} - 17 = \frac{-24}{b + 1}$

In Exercises 87–96, solve each formula for the indicated variable.

87. $h = \frac{1}{2}gt^2; t$
88. $x^2 + y^2 = r^2; x$
89. $h = 64t - 16t^2; t$
90. $y = 16x^2 - 4; x$
91. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; y$
92. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1; x$
93. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1; a$
94. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1; b$
95. $x^2 + xy - y^2 = 0; x$
96. $x^2 - 3xy + y^2 = 0; y$
97. If r_1 and r_2 are the roots of $ax^2 + bx + c = 0$, show that $r_1 + r_2 = -\frac{b}{a}$.
98. If r_1 and r_2 are the roots of $ax^2 + bx + c = 0$, show that $r_1 r_2 = \frac{c}{a}$.

In Exercises 99–100, a stone is thrown upward, higher than the top of a tree. The stone is even with the top of the tree at times t_1 on the way up and t_2 on the way down. If the height of the tree is h, both t_1 and t_2 are solutions of $h = v_0t - 16t^2$.

99. Show that the tree is $16t_1t_2$ feet tall.
100. Show that v_0 is $16(t_1 + t_2)$ feet per second.

We can solve this quadratic equation with the quadratic formula.

$$q = \frac{-\mathbf{50} \pm \sqrt{\mathbf{50}^2 - 4(\mathbf{3})(-\mathbf{248})}}{2(\mathbf{3})}$$

$$q = \frac{-50 \pm \sqrt{2500 + 2976}}{6}$$

$$q = \frac{-50 \pm \sqrt{5476}}{6}$$

$$q = \frac{-50 \pm 74}{6}$$

$$q = \frac{-50 + 74}{6} = \frac{24}{6} = 4 \quad \text{or} \quad q = \frac{-50 - 74}{6} = \frac{-124}{6} = -\frac{62}{3}$$

Because the number of riders cannot be negative, the result of $-\frac{62}{3}$ must be discarded. To generate \$10,208 in daily revenues, the company should raise the fare by 4 quarters, or \$1, to \$11. ■

2.4 EXERCISES

In Exercises 1–34, solve each word problem. ***You may use a calculator if it is helpful.***

1. ***Number problem*** The product of two consecutive even natural numbers is 48. Find the numbers.

2. ***Number problem*** The product of the first and last of three consecutive odd integers is 45. Find the sum of the three integers.

3. ***Geometric problem*** A rectangle is 4 feet longer than it is wide. If its area is 32 square feet, find its dimensions.

4. ***Geometric problem*** A rectangle is 5 times as long as it is wide. If the area is 125 square feet, find its perimeter.

5. ***Geometric problem*** The side of a square is 4 centimeters shorter than the side of a second square. If the sum of their areas is 106 square centimeters, find the length of one side of the larger square.

6. ***Geometric problem*** The base of a triangle is one-third as long as its height. If the area of the triangle is 24 square meters, how long is its base?

7. ***Metal fabrication*** A piece of tin, 12 inches on a side, is to have four equal squares cut from its corners, as in Illustration 1. If the edges are then to be folded up to make a box with a floor area of 64 square inches, find the depth of the box.

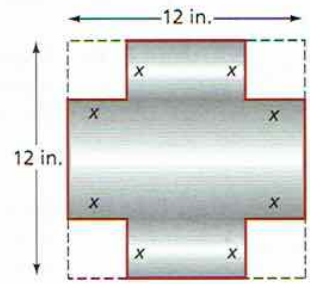

ILLUSTRATION 1

8. ***Making gutters*** A piece of sheet metal, 18 inches wide, is bent to form the gutter shown in Illustration 2. If the cross-sectional area is 36 square inches, find the depth of the gutter.

9. ***Cycling rates*** A cyclist rides from DeKalb to Rockford, a distance of 40 miles. His return trip takes 2 hours longer because his speed decreases by 10 miles per hour. How fast does he ride each way?

ILLUSTRATION 2

10. ***Travel time*** A farmer drives a tractor from one town to another, a distance of 120 kilometers. He drives 10 kilometers per hour faster on the return trip, cutting 1 hour off the time. How fast does he drive each way?

11. ***Uniform-motion problem*** If the speed were increased by 10 miles per hour, a 420-mile trip would take 1 hour less time. How long does the trip take at the slower speed?

12. ***Uniform-motion problem*** By increasing her usual speed by 25 kilometers per hour, a bus driver decreases the time on a 25-kilometer trip by 10 minutes. Find the usual speed.

13. ***Ballistics*** The height of a projectile fired upward with an initial velocity of 400 feet per second is given by the formula $h = -16t^2 + 400t$, where h is the height in feet and t is the time in seconds. Find the time required for the projectile to return to earth.

14. ***Ballistics*** The height of an object tossed upward with an initial velocity of 104 feet per second is given by the formula $h = -16t^2 + 104t$, where h is the height in feet and t is the time in seconds. Find the time required for the object to return to its point of departure.

15. ***Falling coins*** An object will fall s feet in t seconds, where $s = 16t^2$. How long will it take for a penny to hit the ground if it is dropped from the top of the Sears Tower in Chicago? (*Hint:* The tower is 1454 feet tall.)

16. ***Ballistics*** The height of an object thrown upward with an initial velocity of 32 feet per second is given by the formula $h = -16t^2 + 32t$, where t is the time in seconds. How long will it take the object to reach a height of 16 feet?

17. ***Setting fares*** A bus company has 3000 passengers daily, paying a 25¢ fare. For each nickel increase in fare, the company projects that it will lose 80 passengers. What fare increase will produce $994 in daily revenue?

18. ***Jazz concerts*** A jazz group on tour has been drawing every $1 increase in the $12 ticket price, the average attendance will decrease by 50. At what ticket price will nightly receipts be $5600?

19. ***Concert receipts*** Tickets for the annual symphony orchestra pops concert cost $15, and the average attendance at the concerts has been 1200 persons. Management projects that for each 50¢ decrease in ticket price, 40 more patrons will attend. What decrease in ticket price will result in receipts of $17,280?

20. ***Projecting demand*** The *Vilas County News* earns a profit of $20 per year for each of its 3000 subscribers. Management projects that the profit per subscriber would increase by 1¢ for each additional subscriber over the current 3000. How many subscribers are needed to bring a total profit of $120,000?

21. ***Filling a storage tank*** Two pipes are used to fill a water-storage tank. The first pipe can fill the tank in 4 hours, and the two pipes together can fill the tank in 2 hours less time than the second pipe alone. How long would it take for the second pipe to fill the tank?

22. ***Filling a swimming pool*** A hose can fill a swimming pool in 6 hours. Another hose needs 3 more hours to fill the pool than the two hoses combined. How long would it take the second hose to fill the pool?

23. ***Mowing lawns*** Kristy can mow a lawn in 1 hour less time than her brother Steven. Together they can finish the job in 5 hours. How long would it take Kristy if she worked alone?

24. ***Milking cows*** Working together, Sarah and Heidi can milk the cows in 2 hours. If they work alone, it takes Heidi 3 hours longer than it takes Sarah. How long would it take Heidi to milk the cows alone?

25. ***Geometric problem*** Is it possible for a rectangle to have a width that is 3 units shorter than its diagonal and a length that is 4 units longer than its diagonal?

26. ***Geometric problem*** If two opposite sides of a square are increased by 10 meters and the other sides are decreased by 8 meters, the area of the rectangle that is formed is 63 square meters. Find the area of the original square.

27. ***Investment problem*** Maude and Matilda each have a bank CD. Maude's is $1000 larger than Matilda's, but the interest rate is 1% less. Last year Maude received interest of $280, and Matilda received $240. Find the